

# Spontaneous breaking of nilpotent symmetry in boundary BLG theory

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We exploit boundary term to preserve the supersymmetric gauge invariance of Bagger–Lambert–Gustavsson (BLG) theory. The fermionic rigid BRST and anti-BRST symmetries are studied in linear and non-linear gauges. Remarkably, for Delbourgo-Jarvis-Baulieu-Thierry-Mieg (DJBTM) type gauge the spontaneous breaking of BRST symmetry occurs in the BLG theory. The responsible guy for such spontaneous breaking is ghost-anti-ghost condensation. Further, we discuss the ghost-anti-ghost condensates in the modified maximally Abelian (MMA) gauge in the BLG theory.

## I. INTRODUCTION

Bagger and Lambert [1–3] and Gustavsson [4] proposed a formalism to describe multiple M2-branes and it was found that the generalized Jacobi identity for Lie 3-algebra that generalizes the notion of the Lie algebra is essential to define the action with  $\mathcal{N} = 8$  supersymmetry. The 3-algebras, and, in general,  $n$ -algebras were introduced by V. Filippov [5] however, they were intimately related to the Nambu bracket [6]. A coupling of the BLG theory (a three dimensional field theory) to  $D = 3$ ,  $\mathcal{N} = 8$  conformal supergravity is invoked in [7, 8]. In fact, the conformal supergravity multiplet considered there in the component field formulation contains a dreibein, eight Rarita-Schwinger fields and  $SO(8)$  gauge fields. The local symmetry transformation laws of these fields and an invariant Lagrangian of the coupled theory have been constructed. The Lagrangian contains the conformal supergravity multiplet, and commutators of local on-shell super-transformations.

So far the only explicit example of Lie 3-algebra ever considered for the BLG model is  $\mathcal{A}_4$  which is the  $SO(4)$ -invariant algebra with 4 generators. For a more concrete understanding of such model, we need to study more explicit examples of Lie 3-algebra. With the advent of the BLG model, an alternative method to construct a 5-brane action with the gauge symmetry

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associated with the group of volume preserving diffeomorphisms was proposed in [9–11]. Although some steps have already been undertaken [12, 13], the equivalence of the BLG model to the M5-brane description reported in [14, 15] is still to be verified. The BLG theory has been used for analysing a system of M2-branes ending on a M5-brane, so it is worthwhile to study the BLG theory in presence of a boundary. The supersymmetric theories in presence of a boundary have been studied extensively [16, 17]. However, the multiple branes with a boundary are studied in [18–20]. It is well-known that in a supersymmetric theory, the presence of a boundary breaks the supersymmetry. This is because the boundary obviously breaks translational symmetry and since supersymmetry closes on translations. Incidentally, it has been found that the boundary BLG theory respects the gauge symmetry [21]. The BRST Symmetry for the theory of M2-branes are investigated in [22–25].

On the other hand, the ghost-anti-ghost condensation present in the theories possessing gauge symmetry play an important role [26–30]. In this framework, it was shown that such condensation leads a spontaneous breaking of supersymmetries present in the theory. In fact, for non-Abelian theories in the maximally Abelian (MA) gauge the ghost-anti-ghost condensation offers a mechanism to provide the masses of off-diagonal gluons and off-diagonal ghosts [31, 32]. This mechanism gets relevance in infrared Abelian dominance [33], which justifies the dual superconductor picture [34–36] of QCD vacuum for explaining quark confinement [32, 37–39]. The breaking of spontaneous breaking of supersymmetries have led many interesting consequences in different situations [26, 40–43]. Recently, the presence of ghost-anti-ghost condensation has been occurred in ABJM theory [44]. This provides a platform to perform similar investigation in the BLG theory with a Nambu-Poisson 3-bracket which describe the theory of multiple M2-branes.

In this work, we review the supersymmetry in presence of a boundary for simple supersymmetric theory. Further, we recapitulate the BLG theory in presence of boundary condition where we show that only half of the supersymmetry can be preserved by adding a boundary term to the bulk. Since BLG theory on the boundary admits a new gauge symmetry structure based on the 3-algebra, therefore, to quantize such theory we choose a particular gauge. We choose DJBTM type and Landau type gauges in view of their common uses. We show that the quantum actions corresponding to these gauges admit supersymmetric BRST invariance. Further we report the existence of non-vanishing ghost-anti-ghost condensates appeared in DJBTM gauge which leads

to spontaneous supersymmetry breaking. Due to these non-vanishing ghost-anti-ghost condensates the non-linear bosonic fields possess the non-vanishing vacuum expectation values (VEVs). The occurrence of ghost-anti-ghost condensates in MMA gauge is also shown in the boundary BLG theory. Hence, the ghost and anti-ghost fields appeared in condensates are shown as the Nambu-Goldstone particles. We derive the effective potential for the BLG theory which confirms the occurrence of ghost-anti-ghost condensation and hence the spontaneous symmetry breaking in the theory.

The paper is organized as follows. In Sec. II, we give brief review of supersymmetry in presence of a boundary. The BLG theory in presence of boundary is studied in Sec. III. In Sec. IV, we analyse the BRST invariance of the different gauge-fixed actions. The effective potential corresponding to non-linear gauge and the spontaneous breaking of BRST symmetry due to ghost-anti-ghost condensation is presented in Sec. V. Similar investigation for modified MA is reported in Sec. VI. In the last section the results and future investigations are given.

## II. SUPERSYMMETRY IN PRESENCE OF A BOUNDARY

In this section, we analyse the possession of half of the supersymmetry, without introducing explicit boundary conditions [45–47]. We achieve this by adding a boundary term to the bulk Lagrangian of the original theory where the supersymmetric transformations of this boundary term exactly compensates the boundary piece emerged from the supersymmetric transformation of the bulk Lagrangian. We begin with the following simple supersymmetric Lagrangian written in terms of the  $\mathcal{N} = 1$  superfield defined in three dimensions as  $\Phi(\theta) = p + q\theta + r\theta^2$ , where  $\theta$  is a two component Grassmann parameter,

$$\mathcal{L} = D^2[\Phi]_{\theta=0}, \quad (1)$$

where  $D^2 = D^a D_a / 2$  and  $D_a = \partial_a + (\gamma^\mu \theta)_a \partial_\mu$ . The  $\mathcal{N} = 1$  supersymmetric transformations are generated by charge  $Q_a$  as follows

$$\delta\Phi(\theta) = \epsilon^a Q_a \Phi(\theta), \quad (2)$$

where charge is defined by

$$Q_a = \partial_a - (\gamma^\mu \theta)_a \partial_\mu. \quad (3)$$

This supersymmetric transformation for the component fields is thus given by

$$\begin{aligned} \delta p &= \epsilon^a q_a, \\ \delta q_a &= -\epsilon_a r + (\gamma^\mu \epsilon)_a \partial_a p, \\ \delta r &= \epsilon^a (\gamma^\mu \partial_\mu)_a^b q_b. \end{aligned} \quad (4)$$

We remark that the Lagrangian  $\mathcal{L}$  given by Eq. (1) on a manifold without boundaries remains invariant under these supersymmetric transformations, however, in presence of a boundary (say at  $x_3 = 0$ ) this Lagrangian under the supersymmetric transformations leads a total derivative

$$\delta \mathcal{L} = -\partial_3 (\epsilon \gamma^3 q), \quad (5)$$

which leads a boundary term. This, therefore, justifies the statement that braking of the supersymmetry occurs in presence of boundary. To preserve at least a part of the supersymmetry, we can add a boundary term to the theory, such that its supersymmetric transformation cancels the boundary piece generated by the supersymmetric transformation of the bulk Lagrangian. Here, we add or subtract the following boundary term to the bulk Lagrangian given in Eq. (1):

$$\mathcal{L}_b = \partial_3 [\Phi(\theta)]_{\theta=0}. \quad (6)$$

to preserve the supersymmetry generated either by boundary supercharges  $\epsilon^- Q_+$  or  $\epsilon^+ Q_-$  [45, 47] by adding or subtracting  $\mathcal{L}_b$  to supersymmetric Lagrangian  $\mathcal{L}$ . Here we note that we can preserve only half of the supersymmetry because we cant preserve the supersymmetries corresponding to  $\epsilon^- Q_+$  and  $\epsilon^+ Q_-$  instantaneously. Now we define the supersymmetric Lagrangian

$$\mathcal{L}^\pm = \mathcal{L} \pm \mathcal{L}_b = (D^2 \mp \partial_3) [\Phi]_{\theta=0}, \quad (7)$$

where  $\mathcal{L}^+$  denotes the Lagrangian which preserves the supersymmetry corresponding to  $\epsilon^- Q_+$ , however,  $\mathcal{L}^-$  preserves the supersymmetry corresponding to  $\epsilon^+ Q_-$ . The decomposition of bulk supercharge  $Q_a$  is given as  $\epsilon^a Q_a = \epsilon^+ Q_- + \epsilon^- Q_+$ , where  $Q_- = Q'_- + \theta_- \partial_3$ , and  $Q_+ = Q'_+ - \theta_+ \partial_3$ , however,  $Q'_\pm = \partial_\pm - \gamma^s \theta_\mp \partial_s$  are the standard supersymmetry generators for the boundary fields where  $s$  is the index for the coordinates along the boundary.

### III. BLG THEORY IN PRESENCE OF A BOUNDARY

In this section we recapitulate the BLG theory in presence of a boundary. To preserve half of the supersymmetry of the BLG theory we need to add a boundary term as discussed in the previous section. After adding the boundary term the BLG theory still remains gauge variant. To retrieve a gauge invariant BLG theory, we need one more term except the boundary term. Incidentally, the gauge fields in the BLG theory follow the Lie 3-algebra. A Lie 3-algebra is simple generalization of Lie algebra which describes a vector space endowed with a trilinear product,  $[T^A, T^B, T^C] = f_D^{ABC} T^D$ , where  $T^A$  are the generators of the Lie 3-algebra. The totally antisymmetric structure constants satisfy the following Jacobi identity,  $f_G^{[ABC} f_H^{D]EG} = 0$ . In the quantization of BLG theory it is also useful to define the following structure constant,  $C_{EF}^{AB,CD} = 2f_{[E}^{AB[C} \delta_{F]}^D]$ , which is anti-symmetric in the pair of indices and satisfies the following Jacobi identity:  $C_{EF}^{AB,CD} C_{KL}^{GH,EF} + C_{EF}^{GH,AB} C_{KL}^{CD,EF} + C_{EF}^{CD,GH} C_{KL}^{AB,GH} = 0$ . The BLG theory on manifolds without boundaries preserves  $\mathcal{N} = 8$  supersymmetry. However, we perform the analysis for the BLG theory on manifolds with boundaries which preserves  $\mathcal{N} = 1$  supersymmetry generated by the charge  $Q_a = \partial_a - (\gamma^\mu \partial_\mu \theta)_a$ . To write the Lagrangian for BLG theory possessing the boundary term, we first define super-covariant derivatives for matter fields  $X_A, X_A^\dagger$  and the spinor field  $\Gamma_{AB}^a$  as follows,

$$\begin{aligned} \nabla_a X_A^I &= D_a X_A^I - i f_A^{BCD} \Gamma_{aBC} X_D^I, \\ \nabla_a X_A^{I\dagger} &= D_a X_A^{I\dagger} + i f_A^{BCD} X_D^{I\dagger} \Gamma^{aBC}, \end{aligned} \quad (8)$$

$$(\nabla_a \Gamma_b)_{AB} = D_a \Gamma_{bAB} + C_{AB}^{CD,EF} \Gamma_{CDa} \Gamma_{bEF}, \quad (9)$$

where super-derivative  $D_a = \partial_a + (\gamma^\mu \partial_\mu)_a^b \theta_b$ . The Lagrangian for the BLG theory on a manifold without a boundary is defined by

$$\begin{aligned} \mathcal{L} = & \nabla^2 \left[ \frac{k}{4\pi} f^{ABCD} \Gamma_{AB}^a \Omega_{aCD} + \frac{1}{4} (\nabla^a X^I)^A (\nabla_a X_I^\dagger)_A \right. \\ & \left. - \frac{2\pi}{k} \epsilon_{IJKL} f^{ABCD} X_A^I X_B^{K\dagger} X_C^J X_D^{L\dagger} \right]_{\theta=0}, \end{aligned} \quad (10)$$

where

$$\begin{aligned} \Omega_{aAB} &= \omega_{aAB} - \frac{1}{3} C_{AB}^{CD,EF} [\Gamma^{bCD}, \Gamma_{abEF}] \\ \omega_{aAB} &= \frac{1}{2} D^b D_a \Gamma_{bAB} - i C_{AB}^{CD,EF} [\Gamma_{CD}^b, D_b \Gamma_{aEF}] \end{aligned} \quad (11)$$

$$-\frac{1}{3}C_{AB}^{CD,EF}C_{EF}^{GH,IJ}[\Gamma_{CD}^b, \{\Gamma_{bGH}, \Gamma_{aIJ}\}], \quad (12)$$

$$\Gamma_{abAB} = -\frac{i}{2} \left[ D_{(a}\Gamma_{b)AB} - 2iC_{AB}^{CD,EF} \{\Gamma_{aCD}, \Gamma_{bEF}\} \right]. \quad (13)$$

The matter fields  $X_A, X_A^\dagger$  and the spinor field  $\Gamma_{AB}^a$  transform under gauge transformations as follows,  $\Gamma_a \rightarrow iu \nabla_a u^{-1}$ ,  $X^I \rightarrow u X^I$ ,  $X^{I\dagger} \rightarrow X^{I\dagger} u^{-1}$ , where  $X^I = X_A^I T^A$ ,  $X^{I\dagger} = X_A^{I\dagger} T^A$ ,  $\Gamma_a = \Gamma_{aAB} T^A T^B$ .

Now exploiting the a projection operator,  $(P_\pm)_a^b = (\delta_a^b \pm (\gamma^3)_a^b)/2$  the super-covariant derivatives is projected by  $\nabla_{\pm b} = (P_\pm)_b^a \nabla_a$ . Under such projection operator the charge  $Q_a$  splits up into  $Q_{\pm b} = (P_\pm)_b^a Q_a$ . For a boundary fixed at  $x_3$ , coordinate  $\mu$  splits into  $\mu = (\mu, 3)$ , and as a result only half of the supersymmetry can be preserved. The matter and spinor fields on the boundary is represented by  $X'_A, X'^{\dagger}_A$  and  $\Gamma_{AB}^a$ , respectively. Furthermore, the super-covariant derivative for the fields on boundary, is written by  $\nabla'_a$ . On the boundary superfield  $v$  reads  $v'$ . The Lagrangian for the BLG theory having fixed boundary which is invariant under supersymmetry generated by charge  $Q_+$  is given by

$$\mathcal{L}_{sg} = -\nabla'_+ [\mathcal{CS}(\Gamma^v) + \mathcal{M}(X^I, X^{I\dagger}) + \mathcal{K}'(\Gamma', v')]|_{\theta_-=0}, \quad (14)$$

where

$$\begin{aligned} \mathcal{CS}(\Gamma^v) &= \frac{k}{4\pi} \nabla_- [f^{ABCD} \Gamma_{AB}^a \Omega_{aCD}]_{\theta_+=0}, \\ \mathcal{M}(X^I, X^{I\dagger}) &= \frac{1}{4} \nabla_- [(\nabla^a X^I)^A (\nabla_a X_I^\dagger)_A]_{\theta_+=0} \\ &\quad - \frac{2\pi}{k} \nabla_- [\epsilon_{IJKL} f^{ABCD} X_A^I X_B^{K\dagger} X_C^J X_D^{L\dagger}]_{\theta_+=0}, \\ \mathcal{K}'(\Gamma', v') &= -\frac{k}{2\pi} [f_{ABCD} (v'^{-1} \nabla'_+ v')^{AB} (v'^{-1} \mathcal{D}'_- v')^{CD}], \end{aligned} \quad (15)$$

$\Gamma_a^v$  denotes the gauge transformation of  $\Gamma_a$  generated by  $v$ .

It may be noted that the difference  $\mathcal{CS}(\Gamma^v) - \mathcal{CS}(\Gamma) = \mathcal{S}(\Gamma', v')$  defines the boundary potential. So, the total potential of the theory reads  $\mathcal{CS}(\Gamma^v) = \mathcal{CS}(\Gamma) + \mathcal{S}(\Gamma', v')$ . In the absence of coupling between the gauge and the bulk fields this reduces to a potential term for the supersymmetric Wess-Zumino-Witten models

$$\begin{aligned} \nabla'_+ \mathcal{S}(\Gamma', v') &= -\frac{k}{2\pi} \nabla'_+ C_{AB}^{CD,EF} \left[ [(v^{-1'} \mathcal{D}'_- v')^{AB}, (v^{-1'} \mathcal{D}'_3 v')_{CD}] \right. \\ &\quad \left. \times (v^{-1'} \nabla'_+ v')_{EF} \right]_{\theta_-=0}. \end{aligned} \quad (16)$$

#### IV. THE GAUGE-FIXED BOUNDARY BLG THEORY

The gauge invariance of the BLG theory on the boundary ponders the presence of spurious degrees of freedom in the theory. Consequently, we cannot quantize it without fixing a gauge. In this case, therefore, we make the following choice of gauge fixing condition,

$$G \equiv D^a \Gamma_{aAB} = 0. \quad (17)$$

Here if the bulk fields respect some boundary conditions then the path integral must be a sum over the bulk fields obeying those boundary conditions. However, by including the boundary fields in the path integral, both the bulk and the boundary fields will be integrated over. Henceforth, the bulk fields can be split into a pure bulk component and a boundary component. This can be achieved by first including the separate boundary fields and then introducing the Lagrange multipliers to constrain those boundary fields to match the boundary limits of the bulk fields. Therefore, we need separate gauge-fixing terms for the bulk and boundary fields in the BLG theory on a boundary. The boundary gauge-fixing condition can be constructed in such a way that it will lead to the boundary limit of the bulk gauge-fixing condition. Keeping these points in mind, the gauge-fixing condition (17) can be incorporated in the BLG theory at quantum level by adding the following term in the invariant Lagrangian:

$$\mathcal{L}_{gf} = \nabla_+ \nabla_- \left[ f^{ABCD} b_{AB} D^a \Gamma_{aCD} + \frac{\alpha}{2} f^{ABCD} b_{AB} b_{CD} \right]_{\theta=0}. \quad (18)$$

The induced Faddeev-Popov ghost term for this gauge fixing term is written by

$$\mathcal{L}_{gh} = i \nabla_+ \nabla_- \left[ f^{ABCD} \bar{c}_{AB} D^a \nabla_a c_{CD} \right]_{\theta=0}. \quad (19)$$

Incorporating these terms the total Lagrangian density for the boundary BLG theory in Lorentz type gauge is given by

$$\mathcal{L}_{BLG} = \mathcal{L}_{sg} + \mathcal{L}_{gf} + \mathcal{L}_{gh}. \quad (20)$$

This Lagrangian density for BLG theory on boundary remains invariant under the following BRST transformations:

$$s \Gamma_{aAB} = \nabla_a c_{AB}, \quad s c_{AB} = -\frac{1}{2} C_{AB}^{EF, GH} c_{EF} c_{GH},$$

$$\begin{aligned}
s \bar{c}_{AB} &= i b_{AB}, \quad s X_A^{I\dagger} = -i X^{IB\dagger} c_{AB}, \\
s b_{AB} &= 0, \quad s X_A^I = i c_{AB} X^{IB}, \\
s v_{AB} &= -i C_{AB}^{EF, GH} v_{EF} c_{GH}.
\end{aligned} \tag{21}$$

This is also invariant under the another set of supersymmetric transformations called as the anti-BRST transformations which are given by

$$\begin{aligned}
\bar{s} \Gamma_{aAB} &= \nabla_a \bar{c}_{AB}, \quad \bar{s} \bar{c} = -\frac{1}{2} \frac{1}{2} C_{AB}^{EF, GH} \bar{c}_{EF} \bar{c}_{GH}, \\
\bar{s} X_A^{I\dagger} &= -i X^{IB\dagger} \bar{c}_{AB}, \quad \bar{s} \bar{b}_{AB} = 0, \\
\bar{s} X_A^I &= i \bar{c}_{AB} X^{IB}, \quad \bar{s} c_{AB} = i \bar{b}_{AB}, \\
\bar{s} v_{AB} &= -i C_{AB}^{EF, GH} v_{EF} \bar{c}_{GH},
\end{aligned} \tag{22}$$

where new auxiliary field is expressed in terms of original fields as follows,

$$\bar{b}_{AB} = -b_{AB} + i C_{AB}^{EF, GH} c_{EF} \bar{c}_{GH}. \tag{23}$$

This defines the Curci-Ferrari (CF) type restriction.

In the particular limit,  $\alpha = 0$ , the Lorentz gauge corresponds to the Landau gauge, and in this scenario the sum of the gauge fixing and the ghost terms can be written as both the BRST and anti-BRST exact terms

$$\begin{aligned}
\mathcal{L}_{gf} + \mathcal{L}_{gh} &= \frac{i}{2} \nabla_+ \nabla_- s \bar{s} \left[ f^{ABCD} \Gamma_{AB}^a \Gamma_{aCD} \right]_{\theta=0}, \\
&= -\frac{i}{2} \nabla_+ \nabla_- \bar{s} s \left[ f^{ABCD} \Gamma_{AB}^a \Gamma_{aCD} \right]_{\theta=0}.
\end{aligned} \tag{24}$$

Here we note that the BRST or the anti-BRST transformations of the original BLG theory produce a surface term which is compensated by the BRST or the anti-BRST variations of the boundary term of the modified BLG theory. In this way, the overall BRST and the anti-BRST invariances of the BLG theory on boundary are recovered.

Now, we analyse the BLG theory on boundary in DJBTM (non-linear) gauge. In the DJBTM gauge, the sum of the gauge-fixing and ghost terms of the effective Lagrangian is given by

$$\begin{aligned}
\mathcal{L}_g^{DJ} &= \nabla_+ \nabla_- \left[ f^{ABCD} b_{AB} D^a \Gamma_{aCD} + \frac{\alpha}{2} f^{ABCD} b_{AB} b_{CD} + i f^{ABCD} \bar{c}_{AB} D^a \nabla_a c_{CD} \right. \\
&\quad \left. - i \frac{\alpha}{2} C_{AB}^{EF, GH} c_{EF} \bar{c}_{GH} b^{AB} + \frac{\alpha}{8} C_{AB}^{EF, GH} C_{IJ, KL}^{AB} \bar{c}_{EF} \bar{c}_{GH} c^{IJ} c^{KL} \right]_{\theta=0}.
\end{aligned} \tag{25}$$



Further, it can be expressed by

$$\begin{aligned} \mathcal{L}_g^{DJ} = & \nabla_+ \nabla_- \left[ f^{ABCD} b_{AB} D^a \Gamma_{aCD} + \frac{\alpha}{2} f^{ABCD} b_{AB} b_{CD} + i f^{ABCD} \bar{c}_{AB} D^a \nabla_a c_{CD} \right. \\ & \left. - i \frac{\alpha}{2} C_{AB}^{EF, GH} c_{EF} \bar{c}_{GH} b^{AB} - \frac{\alpha}{4} C_{AB}^{EF, GH} C_{IJ, KL}^{AB} c_{EF} \bar{c}_{GH} c^{IJ} \bar{c}^{KL} \right]_{\theta=0}. \end{aligned} \quad (26)$$

To scrutinize the non-zero gauge parameter, it is expressed by

$$\begin{aligned} \mathcal{L}_g^{DJ} = & \nabla_+ \nabla_- \left[ \frac{\alpha}{2} \left( b_{AB} - \frac{i}{2} C_{AB}^{EF, GH} c_{EF} \bar{c}_{GH} + \frac{1}{\alpha} D_a \Gamma_{AB}^a \right)^2 - \frac{1}{2\alpha} (D_a \Gamma_{AB}^a)^2 \right. \\ & \left. + i f^{ABCD} \bar{c}_{AB} D^a \nabla_a c_{CD} - \frac{\alpha}{8} C_{AB}^{EF, GH} C_{IJ, KL}^{AB} c_{EF} \bar{c}_{GH} c^{IJ} \bar{c}^{KL} \right]_{\theta=0}. \end{aligned} \quad (27)$$

The total Lagrangian density for BLG theory on boundary in DJBTM gauge is written as the sum of invariant part and the gauge-fixed part,

$$\mathcal{L}'_{BLG} = \mathcal{L}_{sg} + \mathcal{L}_g^{DJ}, \quad (28)$$

which remains invariant under the BRST and anti-BRST transformations given respectively in (21) and (22). The gauge-fixed part of the above Lagrangian density  $\mathcal{L}_g^{DJ}$  can be expressed as the BRST and anti-BRST exact terms as follows,

$$\begin{aligned} \mathcal{L}_g^{DJ} &= \frac{i}{2} \nabla_+ \nabla_- s \bar{s} \left[ f^{ABCD} (\Gamma_{AB}^a \Gamma_{aCD} - \alpha \bar{c}_{AB} c_{CD}) \right]_{\theta=0}, \\ &= -\frac{i}{2} \nabla_+ \nabla_- \bar{s} s \left[ f^{ABCD} (\Gamma_{AB}^a \Gamma_{aCD} - \alpha \bar{c}_{AB} c_{CD}) \right]_{\theta=0}. \end{aligned} \quad (29)$$

The non-linear auxiliary field  $b$  plays an important role as an order parameters in analysing the spontaneous breaking of BRST symmetry.

## V. EFFECTIVE POTENTIAL FOR BOUNDARY BLG THEORY IN NON-LINEAR GAUGE

In this section, we investigate the spontaneous breakdown of BRST supersymmetry in boundary BLG theory. In this context, we first define the potential  $V(b)$  for multiplier fields  $b_{AB}$  as follows

$$V(b) = \nabla_+ \nabla_- \left[ -\frac{\alpha}{2} \left( b_{AB} - \frac{i}{2} C_{AB}^{EF, GH} c_{EF} \bar{c}_{GH} + \frac{1}{\alpha} D_a \Gamma_{AB}^a \right)^2 \right]_{\theta=0}. \quad (30)$$

From this expression it is evident that the potential has extremum for gauge parameter  $\alpha$  at

$$b_{AB} = \frac{i}{2} C_{AB}^{EF, GH} c_{EF} \bar{c}_{GH} - \frac{1}{\alpha} D_a \Gamma_{AB}^a. \quad (31)$$

The vacuum expectation value of non-linear bosonic field  $b$  corresponding to the vanishing spinor expectation value  $\langle \Gamma_{AB}^a \rangle = 0$  takes

$$\langle 0 | b_{AB} | 0 \rangle = \frac{1}{2} \langle 0 | i C_{AB}^{EF, GH} c_{EF} \bar{c}_{GH} | 0 \rangle. \quad (32)$$

In presence of ghost-anti-ghost condensation

$$\langle 0 | i C_{AB}^{EF, GH} c_{EF} \bar{c}_{GH} | 0 \rangle \neq 0, \quad (33)$$

the non-linear field  $b_{AB}$  acquires the non-vanishing vacuum expectation value (VEV), i.e.,

$$\langle 0 | b_{AB} | 0 \rangle = \frac{1}{2} \langle 0 | i C_{AB}^{EF, GH} c_{EF} \bar{c}_{GH} | 0 \rangle \neq 0. \quad (34)$$

As a result, the spontaneous breaking in BRST symmetry occurs due to this non-vanishing VEV,

$$\langle 0 | s \bar{c}_{AB} | 0 \rangle = \langle 0 | i b_{AB} | 0 \rangle = -\frac{1}{2} \langle 0 | C_{AB}^{EF, GH} c_{EF} \bar{c}_{GH} | 0 \rangle \neq 0. \quad (35)$$

Exploiting CF type condition (23), we observe that the spontaneous breaking of anti-BRST symmetry also occurs,

$$\langle 0 | \bar{s} c_{AB} | 0 \rangle = \langle 0 | i \bar{b}_{AB} | 0 \rangle = -\frac{1}{2} \langle 0 | C_{AB}^{EF, GH} c_{EF} \bar{c}_{GH} | 0 \rangle \neq 0. \quad (36)$$

The spontaneous breaking of BRST and anti-BRST symmetries reflect the presence of massless Nambu-Goldstone particles for boundary BLG theory following Nambu-Goldstone theorem. Here the ghosts and anti-ghosts are Nambu-Goldstone particles.

To specify whether such ghost-anti-ghost condensations and therefore spontaneous symmetry breaking take place or not, it is important to evaluate the effective potential for the composite operator  $i C_{AB}^{EF, GH} c_{EF} \bar{c}_{GH}$ . Performing the analysis for Lie 3-algebra as in [26] leads to the total bosonic effective potential

$$V(b, \phi) = V(\phi) + \nabla_+ \nabla_- \left[ -\frac{\alpha}{2} \left( b_{AB} + \frac{1}{2\alpha} \phi_{AB} \right)^2 \right]_{\theta=0}, \quad (37)$$

where  $V(\phi)$  refers the effective potential for  $\phi_{AB} \sim -\alpha \langle 0 | i C_{AB}^{EF,GH} c_{EF} \bar{c}_{GH} | 0 \rangle$ . Now we see that the potential  $V(\phi)$  has stationary points for  $b_{AB} = -\frac{1}{2\alpha} \phi_{AB}$ . Hence the condensation is meaningful for  $\alpha > 0$  only. However, for Landau gauge condition ( $\alpha = 0$ ) where gauge parameter takes zero value the minimum or maximum of the potential of the field  $b_{AB}$  occurs at  $b_{AB} = 0$ , which implies  $\langle 0 | i \bar{b}_{AB} | 0 \rangle = 0$ . Therefore, in the Landau gauge the spontaneous BRST symmetry breaking due to the mechanism mentioned above can not occur (at least in the tree level).

## VI. EFFECTIVE POTENTIAL FOR BOUNDARY BLG THEORY IN MMA GAUGE

In this section, we discuss ghost-anti-ghost condensation for boundary BLG theory in MMA gauge [48, 49]. To do so, let us begin by decomposing the spinor field in diagonal and off-diagonal components as follows

$$\Gamma_{AB}^a = \gamma_{aAB}^i T_i + \Upsilon_{aAB}^\alpha T_\alpha, \quad (38)$$

where  $T_i \in \mathcal{H}$  and  $T_\alpha \in \mathcal{G} - \mathcal{H}$ . Here  $\mathcal{H}$  refers to the Cartan subalgebra of the Lie algebra  $\mathcal{G}$ . Now, the gauge-fixed Lagrangian density in MA gauge incorporating diagonal and off-diagonal decomposition is given by

$$\begin{aligned} \mathcal{L}_g^{MA} = & -i \nabla_+ \nabla_- s \left[ f^{ABCD} \bar{c}_{AB} \left\{ \nabla_a [\gamma] \Upsilon_{CD}^a + \frac{\alpha}{2} b_{AB} \right\} - i \frac{\zeta}{2} C_{AB}^{EF,GH} \bar{c}^{AB} \bar{c}_{EF} c_{GH} \right. \\ & \left. - i \frac{\zeta}{4} C_{AB}^{EF,GH} c^{AB} \bar{c}_{EF} \bar{c}_{GH} \right]_{\theta=0}, \end{aligned} \quad (39)$$

which can further be expanded by assigning the BRST transformation on the superfields as follows

$$\begin{aligned} \mathcal{L}_g^{MA} = & \nabla_+ \nabla_- \left[ f^{ABCD} b_{AB} \nabla_a [\gamma] \Upsilon_{CD}^a + \frac{\alpha}{2} f^{ABCD} b_{AB} b_{CD} \right. \\ & + i f^{ABCD} \bar{c}_{AB} \nabla_a [\gamma] \nabla^a [\gamma] c_{CD} - i C_{AB}^{EF,GH} C_{IJ,KL}^{AB} \bar{c}_{EF} \Upsilon_{aGH} c^{IJ} \Upsilon^{aKL} \\ & + i C_{AB}^{EF,GH} \bar{c}^{AB} \nabla_a [\gamma] (\Upsilon_{EF}^a c_{GH}) + i C_{AB}^{EF,GH} \bar{c}^{AB} \nabla_a [\gamma] \Upsilon_{EF}^a c_{GH} \\ & + \frac{\zeta}{8} C_{AB}^{EF,GH} C_{IJ,KL}^{AB} \bar{c}_{EF} \bar{c}_{GH} c^{IJ} c^{KL} + \frac{\zeta}{4} C_{AB}^{EF,GH} C_{IJ,KL}^{AB} \bar{c}_{EF} \bar{c}_{GH} c^{IJ} c^{KL} \\ & + i \frac{\zeta}{2} C_{AB}^{EF,GH} c^{AB} b_{EF} \bar{c}_{GH} - i \zeta C_{AB}^{EF,GH} b^{AB} \bar{c}_{EF} c_{GH} \\ & \left. + \frac{\zeta}{4} C_{AB}^{EF,GH} C_{IJ,KL}^{AB} \bar{c}_{EF} \bar{c}_{GH} c^{IJ} c^{KL} \right]_{\theta=0}. \end{aligned} \quad (40)$$

The requirement of the orthosymplectic invariance of the MMA gauge yields the quartic ghost interaction as  $\zeta = \alpha$ . Therefore, the above expression in the MMA gauge reduces to

$$\begin{aligned} \mathcal{L}_g^{MMA} = & \nabla_+ \nabla_- \left[ \frac{\alpha}{2} \left( b_{AB} - iC_{AB}^{EF, GH} \bar{c}_{EF} c_{GH} + \frac{1}{\alpha} \nabla_a [\gamma] \Upsilon_{AB}^a \right)^2 - \frac{1}{2\alpha} (\nabla_a [\gamma] \Upsilon_{AB}^a)^2 \right. \\ & - iC_{AB}^{EF, GH} C_{IJ, KL}^{AB} \bar{c}_{EF} c_{GH} \Upsilon_a^{IJ} \Upsilon^{aKL} + iC_{AB}^{EF, GH} \bar{c}^{AB} \nabla_a [\gamma] (\Upsilon_{EF}^a c_{GH}) \\ & \left. - i f^{ABCD} \bar{c}_{AB} \nabla_a [\gamma] \nabla^a [\gamma] c_{CD} \right]_{\theta=0}. \end{aligned} \quad (41)$$

For the MMA gauge, the potential for non-linear field  $b$  has its extremum at

$$b_{AB} = iC_{AB}^{EF, GH} \bar{c}_{EF} c_{GH} - \frac{1}{\alpha} \nabla_a [\gamma] \Upsilon_{AB}^a. \quad (42)$$

So, the VEV in this case reads

$$\langle 0 | b_{AB} | 0 \rangle = \langle 0 | iC_{AB}^{EF, GH} \bar{c}_{EF} c_{GH} | 0 \rangle - \frac{1}{\alpha} \langle 0 | \nabla_a [\gamma] \Upsilon_{AB}^a | 0 \rangle. \quad (43)$$

One can obtain gauge parameter  $\beta$  dependence of the vacuum-to-vacuum amplitude  $Z$  as following:

$$\frac{\delta Z}{\delta \beta} = \frac{1}{2} \int d^3x \langle 0; \text{out} | s \left( \bar{c}_{AB}(x) b^{AB}(x) \right) | 0; \text{in} \rangle \quad (44)$$

It signifies that the BLG theories with different gauge parameters are different theories. In this case the total bosonic effective potential is computed by

$$V(\bar{b}, b, \phi) = V(\phi) - \frac{\beta}{2} \left( \bar{b} + \frac{1}{\zeta} \phi \right)^2 - \frac{\alpha}{2} b_{AB} b^{AB}, \quad (45)$$

where we have omitted the index of the diagonal component. Here we note that the effective potential  $V(\phi)$  has minima at non-zero values of  $\alpha$ . Therefore, the spontaneous breakdown of the BRST or anti-BRST could happen, if  $\zeta > 0$  and  $\beta < 0$ . Since the total bosonic effective potential has an absolute minimum at non-zero value of  $\bar{b} = -\frac{1}{\zeta} \phi$ . This shows that due to presence of the ghost-anti-ghost condensates the boundary BLG theory in MMA gauge the spontaneous breakdown of the BRST symmetry occurs.

The spontaneous BRST and anti-BRST supersymmetry breaking reflect that the Nambu-Goldstone particles associated with these can be identified as the diagonal anti-ghost or diagonal ghost, respectively. It means that the diagonal ghost and the diagonal anti-ghost are massless particles which are consistent with the infrared Abelian dominance. Since for the infrared Abelian dominance, the off-diagonal components of ghosts become massive while the diagonal components remain massless.

## VII. CONCLUSION

As we know, the basic objects to unify string theories in ten dimensions are M2-branes and M5-branes (the magnetic version of M2-branes in the sense that M5-branes couple to the dual background three form C-field in 11D supergravity). Therefore we could consider that M-branes are the most fundamental objects [50]. To understand the mysterious nature of the M-theory, it is desirable to understand properties of M2-branes and M5-branes. A single M2-brane or a single M5-brane had already been known for quite a long time. However, the multiple M2-branes is studied in recent past years by BLG theory and ABJM theory [1–4, 51]. In fact, the BLG theory has been identified with the M5-brane action in presence of a large C-field [11]. It is obvious that BLG theory by itself cannot be identified with a 6 dimensional theory, as it is a 3 dimensional theory.

In this paper, we analysed the BLG theory which follows the Lie 3-algebra in different gauges. In view of their extreme importance, we choose these gauges to be the Delbourgo-Jarvis and Baulieu-Thierry-Mieg (DJBTM) gauge and modified maximally Abelian (MMA) gauge. The quantum actions corresponding to these gauges admit supersymmetric BRST invariance. We have shown that the existence of non-vanishing ghost-anti-ghost condensates appeared in DJBTM gauge which can also be justified by symmetry breaking considerations. Due to these non-vanishing ghost-anti-ghost condensates the non-linear bosonic fields possess the non-vanishing vacuum expectation values (VEVs). The occurrence of ghost-anti-ghost condensates in MMA gauge is also found in the BLG theory which causes the spontaneous breaking of the BRST symmetry. The ghost and anti-ghost fields involved in the condensates are identified as Nambu-Goldstone particles. The expression for effective potential for the BLG theory is given which confirms the occurrence of ghost-anti-ghost condensation and hence the spontaneous symmetry breaking in the theory. Our present investigation will help in performing the numeric simulations for the propagator for non-linear gauge. Also this analysis will help in understanding the more explicit examples of Lie 3-algebra. Another possible extension of the present work is to explore this in an alternative formalism for boundary supersymmetry involving SIM (1)

superspace [52–54].

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